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Home Work 1

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1 Wave function of object of mass m in a quantum mechanics system.

$$\psi(x,t) = \psi_0 \exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

Determine V(x) of the system:-

solution:-

shrodenger equation:-

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x)$$

$$\frac{\partial\psi}{\partial t} = \psi_0 \frac{-i\hbar}{2mb^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

$$= \frac{-i\hbar\psi_0}{2mb^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

$$\frac{\partial\psi}{\partial x} = \psi_0 \frac{-\cancel{2}x}{\cancel{2}b^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

$$\frac{\partial\psi}{\partial x} = -\psi_0 \frac{x}{b^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = -\psi_0\frac{x}{b^2}\frac{-2x}{2b^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right) + \frac{-\psi_0}{b^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right)$$

$$= \frac{\cancel{2}x^2}{\cancel{2}b^4}\psi_0\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar}{2mb^2}t\right) - \frac{\psi_0}{b^2}\exp\left(\frac{-x^2}{2b^2} - \frac{i\hbar t}{2mb^2}\right)$$

subtitude on shrodenger eq:-

$$\frac{\hbar^2}{2mb^2}\cancel{\psi_0}\,\cancel{\exp\left(\frac{-x^2}{2b^2}-\frac{i\hbar}{2mb^2}t\right)}=\frac{-\hbar^2}{2m}\left(\frac{x^2\cancel{\psi_0}}{b^4}\cancel{\exp\left(-\frac{x^2}{2b^2}-\frac{i\hbar t}{2mb^2}\right)}\right.$$

$$\left.-\frac{\cancel{\psi_0}}{b^2}\cancel{\exp\left(\frac{-x^2}{2b^2}-\frac{i\hbar t}{2mb^2}\right)}\right)+V(x)\cancel{\psi_0}\cancel{\exp\left(\frac{-x^2}{2b}-\frac{i\hbar t}{2mb^2}\right)}$$

$$\cancel{\frac{\hbar^2}{2mb^2}}=\frac{-\hbar^2x^2}{2mb^4}+\cancel{\frac{\hbar^2}{2mb^2}}+V(x)$$

$$\boxed{\frac{\hbar^2x^2}{2mb^4}=V(x)}$$

"some math I use it"

* Fourier integrals :-

+ fourier series.

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos nx + B_n \sin nx.$$

Where.

$$A_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos mx \, dx \quad , m = 0, 1 \ldots$$

$$B_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin mx \, dx \quad , m = 1, 2 \ldots$$

+ fourier expansion:-

$$f(x) = (2\pi)^{-1/2} \sum_{n=-\infty}^{n=\infty} C_n e^{inx}$$

Where :-

$$\cancel{C_n = (2\pi)^{-1/2} \sum_{n=-\infty}^{\infty} C_n e^{im}}$$

$$C_n = (2\pi)^{-1/2} \int_{-\pi}^{\pi} f(x) e^{-imx} dx.$$

$\Rightarrow$ if the function define in some interval $(-L, +L)$ such that $f(x+2L) = f(x)$

$$f(x) = (2\pi)^{-1/2} \sum_{n=-\infty}^{\infty} C_n e^{\frac{in\pi x}{L}}$$

$$C_n = (2\pi)^{-\frac{1}{2}} \int_{-L}^{L} f(x) e^{\frac{-im\pi x}{L}} dx.$$

+ Fourier transformation:-

- if the function with wich we deal are not periodic, but define for all real values of x, $-\infty < x < \infty$

↳ can be also expressed in terms of complex exponential by taking $L \to \infty$

so

$$f(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} C_n e^{\frac{+in x\pi}{L}} dx. \quad , \text{let } k = \frac{n\pi}{L}$$

and let $g(k) = \frac{L C_n}{\pi}$

so

$$f(x) = \int_{-\infty}^{\infty} (2\pi)^{-\frac{1}{2}} g(k) e^{ikx} dk.$$

$$\boxed{f(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} g(k) e^{ikx} dk.} \quad \text{—(1)}$$

AND

$$\boxed{g(k) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx.} \quad \text{—(2)}$$

(1), (2) → fourier transformation.

fourier tansformation of shrodenger equation.

$$\psi(x,t) = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \phi(P_x,t) e^{\frac{iP_x x}{\hbar}} dp.$$

$$\phi(P_x,t) = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \psi(x,t) e^{\frac{-iP_x x}{\hbar}} dx$$

if weuse eq (1)

$$f(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} g(k) e^{ikx} dk$$ and substitude (2) in (1)

$$f(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x') e^{-ikx'} dx' e^{ikx} dk.$$

$\cancel{f(x) = (2\pi)^{-\frac{1}{2}}}$

$$f(x) = (2\pi)^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x') \, e^{ik(x-x')} \, dx' dk.$$

but Dirac delta function

$$\delta(x-x') = (2\pi)^{-1} \int_{-\infty}^{\infty} e^{ik(x-x')} dk. \quad \text{Where} \quad \delta(x-x') = \begin{cases} 1, & x = x' \\ 0, & \text{otherwise} \end{cases}$$

so

$$f(x) = \int_{-\infty}^{\infty} f(x') \, \delta(x-x') \, dx' \qquad \text{so} \quad f(x) = f(x')$$

- if the function convolution of two function f_1 and f_2 is defined as the integral

$$F(x) = \int_{-\infty}^{\infty} f_1(y) \, f_2(x-y) \, dy$$

if $G(k)$ is fourier transform of $F(x)$ and $g_1(k)$ and $g_2(k)$ are the fourier transform of $f_1(x)$ and $f_2(x)$

Then

$$G(k) = (2\pi)^{\frac{1}{2}} g_1(k) \, g_2(k)$$

$\Downarrow$

This is called convolution theorm

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② shrodenger equation:-

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x,t)\, \psi(x,t)$$

where,

$$\psi(x,t) = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \phi(P_x,t)\, e^{\frac{+iP_x}{\hbar}} dp$$

$$\phi(P_x,t) = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \psi(x,t)\, e^{\frac{-iPx}{\hbar}} dx$$

I will derive. $i\hbar \frac{\partial \phi(P_x,t)}{\partial t}$

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = i\hbar \frac{\partial}{\partial t} \left((2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \psi(x,t)\, e^{\frac{-iP_x x}{\hbar}} dx \right)$$

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = i\hbar (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \frac{\partial \psi(x,t)}{\partial t}\, e^{\frac{-iP_x x}{\hbar}} dx$$

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} i\hbar \frac{\partial \psi(x,t)}{\partial t}\, e^{\frac{-iP_x x}{\hbar}} dx$$

where $i\hbar \frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x,t)\, \psi(x,t)$

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = \left(\frac{P^2}{2m} + V(x,t) \right) \psi(x,t)$$

$$\boxed{\begin{aligned} P &= -i\hbar \frac{\partial}{\partial x} \\ E &= i\hbar \frac{\partial}{\partial t} \\ P^2 &= -\hbar^2 \frac{\partial^2}{\partial x^2} \end{aligned}}$$

so

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \left(\frac{p^2}{2m} + V(x,t)\right) \psi(x,t)\, e^{-\frac{ipx}{\hbar}} dx.$$

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = (2\pi\hbar)^{-\frac{1}{2}} \frac{p^2}{2m} \int_{-\infty}^{\infty} \psi(x,t)\, e^{-\frac{ipx}{\hbar}} dx + (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} V(x,t)\, \psi(x,t)\, e^{-\frac{ipx}{\hbar}} dx$$

$\phi(P_x,t)$

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = \frac{p^2}{2m} \phi(P_x,t) + (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} V(x,t)\, \psi(x,t)\, e^{-\frac{ipx}{\hbar}} dx$$

by the convolution theorm.

$F(r) = \int f_1(R)\, f_2(r-R)\, dR \rightarrow$ Then $G(k) = (2\pi)^{-\frac{3}{2}} g_1(k)\, g_2(k)$ on three dimension

$F(x) = \int f_1(y)\, f_2(x-y)\, dy \rightarrow$ Then $G(k) = (2\pi)^{-\frac{1}{2}} g_1(k)\, g_2(k)$ on one dimension

$$\int_{-\infty}^{\infty} (2\pi\hbar)^{-\frac{1}{2}} V(x,t)\, \psi(x,t)\, e^{-\frac{ipx}{\hbar}} dx \overset{\text{like}}{=} \int_{-\infty}^{\infty} G(k)\, e^{-\frac{ipx}{\hbar}} dx$$

$$F(x) = \int_{-\infty}^{\infty} V(p-p',t)\, \phi(p',t)\, dp' \longleftarrow F(x)$$

So the equation become.

$$i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = \frac{p^2}{2m} \phi(P_x,t) + \int_{-\infty}^{\infty} V(p-p',t)\, \phi(p',t)\, dp'$$

but

$$V(p-p',t) = (2\pi\hbar)^{-1} \int e^{-\frac{i(p-p')x}{\hbar}} V(x,t)\, dx.$$

by dirac delta function.

in general $\leftarrow$ $\delta(x-x') = (2\pi)^{-1} \int e^{ik(x-x')} dk$

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but in general the shrodenger eq at momentum space.

$$\boxed{i\hbar \frac{\partial \phi(P_x,t)}{\partial t} = \frac{P^2}{2m}\phi(P_x,t) + \int_{-\infty}^{\infty} V(P-P')(t)\,\phi(P',t)\,dP'}$$

Prove that the solution in the momentum space normalized of the solution in position space is normalized.

we know

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$$

$$\text{but } \psi(x,t) = (2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \phi(P_x,t)\, e^{\frac{+iP_x x}{\hbar}} dP_x$$

$$\text{so } \int_{-\infty}^{\infty} \psi^*(x,t)\, \psi(x,t)\, dx$$

$$= \int_{-\infty}^{\infty} \left((2\pi\hbar)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \phi^*(P_x',t)\, e^{\frac{-iP_x' x}{\hbar}} dP_x' \right) \left((2\pi\hbar)^{-\frac{1}{2}} \int \phi(P_x,t)\, e^{\frac{iP_x x t}{\hbar}} dP_x \right) dx$$

$$= \int_{-\infty}^{\infty} (2\pi\hbar)^{-1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi^*(P_x',t)\, \phi(P_x,t)\, e^{\frac{ix}{\hbar}(P_x - P_x')t}\, dP_x\, dP_x'\, dx$$

$$\text{but } \delta(P - P_x') = (2\pi\hbar)^{-1} \int_{-\infty}^{\infty} e^{\frac{ix}{\hbar}(P_x - P_x')t} dx = \begin{cases} 0, & P_x \neq P_x' \\ 1, & P_x = P_x' \end{cases}$$

$$\text{so } \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi^*(P_x',t)\, \phi(P_x,t)\, \delta(P_x - P_x')\, dP_x\, dP_x'$$

$$\delta(P_x - P_x') = 1 \text{ if } P_x = P_x'$$

$$\text{so } \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = \int_{-\infty}^{\infty} \phi^*(P_x,t)\, \phi(P_x,t)\, dP_x = 1$$

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = \int_{-\infty}^{\infty} |\phi(P_x,t)|^2 dP_x = 1 \qquad \#$$

d proved.

so $\int_{-\infty}^{\infty} |\phi(p_x,t)|^2 \, dp_x = 1$ is also normalize.

③ Infinite square Well.

$$V(x) = \begin{cases} 0 \ , & -\frac{a}{2} \leq x \leq \frac{a}{2} \\ \infty \ , & \text{otherwise} \end{cases}$$

shrodinger equation.

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x,t)\Psi$$

by seperation variable.

$$\Psi(x,t) = \psi(x)\,\phi(t)$$

$$\frac{\partial \Psi(x,t)}{\partial t} = \psi \frac{d\phi}{dt} \ , \quad \frac{\partial^2 \Psi(x,t)}{\partial x^2} = \phi \frac{d^2\psi(x)}{dx^2}$$

so

$$\frac{i\hbar \psi \frac{d\phi}{dt} = \frac{-\hbar^2}{2m} \phi \frac{d^2\psi}{dx^2} + V(x)\psi(x)\phi(t)}{\psi(x)\,\phi(t)}$$

$$\underbrace{\frac{i\hbar}{\phi(t)} \frac{d\phi}{dt}}_{\text{depend only on } t} = \underbrace{\frac{-\hbar^2}{2m} \frac{1}{\psi} \frac{d^2\psi}{dx^2} + V(x)}_{\text{depend only on } x}$$

constant = constant

$$\frac{i\hbar}{\phi(t)} \frac{d\phi}{dt} = E \longrightarrow \frac{d\phi}{dt} = \frac{-iE}{\hbar}\phi(t)$$

$$\text{so } \boxed{\phi(t) = e^{\frac{-iEt}{\hbar}}}$$

time independent shrodinger equation.

$$\left(\frac{-\hbar^2}{2m\psi}\frac{d^2\psi}{dx^2} + V(x) = E\right)\psi(x)$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi(x) = E\,\psi(x)$$

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for infinit square Well
$V=0$ inside it

so the time independent shrodenger eq become-

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi(x) \quad , \text{let } \frac{2mE}{\hbar^2} = k^2$$

$$r^2 + k^2 = 0$$

$$r^2 = -k^2$$

$$r = \pm ik.$$

so the solution.

$$\psi(x) = A\sin kx + B\cos kx.$$

4 the boundary condition:-

$$\psi\left(\frac{a}{2}\right) = \psi\left(\frac{-a}{2}\right) = 0$$

$\psi(x) = A\sin kx + B\cos kx$.

at $x = \frac{-a}{2}$.

$\psi(\frac{-a}{2}) = A\sin -\frac{ka}{2} + B\cos -\frac{ka}{2}$; where $\sin(-x) = -\sin x$
$\cos(-x) = \cos x$

$\psi(\frac{-a}{2}) = -A\sin\frac{ka}{2} + B\cos\frac{ka}{2}$ —(1)

at $x = \frac{a}{2}$,

$\psi(\frac{a}{2}) = A\sin\frac{ka}{2} + B\cos\frac{ka}{2}$ —(2)

$\boxed{\text{※}}$ (1) + (2)

$\psi(\frac{-a}{2}) + \psi(\frac{a}{2})$

$2B\cos\frac{ka}{2} = 0 \Rightarrow$ if we put $B=0$ there is no wave function (reject) so $B \neq 0$.

$\cos\frac{ka}{2} = 0$; $\frac{ka}{2} = \frac{m\pi}{2}$

Where m is odd.

$\boxed{k_m = \frac{m\pi}{a}}$.

$\boxed{*}$ (1) - (2)

$\psi'(-a/2) + \psi'(a/2)$

$2A \sin \frac{ka}{2} \Rightarrow$ if put $A=0$ there is no wave function so it is reject $\Rightarrow$ so $A \neq 0$

$\Rightarrow \sin \frac{ka}{2} = 0$; $\frac{ka}{2} = n\pi$

Where n is any number.

$k_n = \frac{2n\pi}{a}$, let $2n = L$

Where L is even #

$$\boxed{k_L = \frac{L\pi}{a}}$$

we have

$k_L = \frac{L\pi}{a}$, Where L is even.

$k_m = \frac{m\pi}{a}$; Where m is odd.

for $k_L = \frac{L\pi}{a}$ substitude on ①

$\psi(\frac{a}{2}) = A \sin \frac{ka}{2} + B \cos \frac{ka}{2} = 0$

$\psi(a/2) = A \sin \frac{L\pi \cancel{a}}{2\cancel{a}} + B \cos \frac{L\pi \cancel{a}}{2\cancel{a}} = 0$

$\psi(a/2) = A \cancel{\sin \frac{L\pi}{2}}_{0} + B \cos \frac{L\pi}{2} = 0$

Where L is even.

$\psi(a/2) = B \cos L\pi = 0$

so $\boxed{B = 0}$

$B = 0$ when $k_L = \frac{L\pi}{a}$, where L is even.

for $k_m = \frac{m\pi}{a}$ substitude on ②

$\psi(\frac{a}{2}) = A \sin \frac{m\pi \cancel{a}}{2\cancel{a}} + B \cos \frac{m\pi \cancel{a}}{2\cancel{a}} = 0$

$\psi(\frac{a}{2}) = A \sin \frac{m\pi}{2} + B \cancel{\cos \frac{m\pi}{2}}_{0} = 0$

where m is odd

$\psi(\frac{a}{2}) = A \sin \frac{m\pi}{2} = 0$

$A=0$ When $k_m = \frac{m\pi}{a}$, where m is odd.

so we have 2 solution.

① $\psi_{even} = A \sin k_L x = A \sin \frac{L\pi}{a} x$.

② $\psi_{odd} = B \cos k_m x = \cancel{B} \cos \frac{m\pi}{a} x$

$\psi_{in\ general} = A \sin \frac{L\pi}{a} x + B \cos \frac{m\pi}{a} x$; where

L → even L = 2n

m → odd. m = (2n − 1)

$\psi_{in\ general} = \underbrace{A \sin \frac{2n\pi x}{a}}_{\psi_{even}} + \underbrace{B \cos \frac{(2n-1)\pi x}{a}}_{\psi_{odd}}$ n → is any #

By Normalize ψ_{even}

$$\int_{-a/2}^{a/2} |\psi_{even}|^2 = A^2 \int_{-a/2}^{a/2} \sin^2 \frac{2n\pi x}{a}\, dx = \frac{A^2}{2} \int_{-a/2}^{a/2} \left(1 - \cos \frac{4n\pi x}{a}\right) dx = 1$$

$$\frac{A^2}{2} \left(\int_{-a/2}^{a/2} dx - \int_{-a/2}^{a/2} \cos \frac{4n\pi x}{a}\, dx \right) = 1$$

$$\frac{A^2}{2} \left(x - \frac{a \sin(4n\pi x/a)}{4n\pi} \right) \Bigg|_{-a/2}^{a/2} = 1$$

$$= \frac{A^2}{2} \left(\frac{a}{2} - \frac{a}{4n\pi} \cancel{\sin \frac{4n\pi a}{2a}}^{\,0} - \left(\frac{-a}{2} - \frac{a}{4n\pi} \cancel{\sin \frac{-4n\pi a}{2a}}^{\,0} \right) \right)$$

$$\frac{A^2}{2} \frac{2a}{2} = 1$$

$$A^2 \frac{a}{2} = 1$$

$$A^2 = \frac{2}{a} \rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$ #

By Normalize ψ_{odd}.

$$1 = \int_{-a/2}^{a/2} |\psi_{odd}|^2 = \int_{-a/2}^{a/2} B^2 \cos^2 \frac{(2n-1)\pi x}{a}\, dx = \int_{-a/2}^{a/2} \frac{B^2}{2}\left(1 + \cos \frac{2(2n-1)\pi x}{a}\right) dx$$

$$1 = \frac{B^2}{2}\int_{-a/2}^{a/2} 1\, dx + \frac{B^2}{2}\int_{-a/2}^{a/2} \cos \frac{2(2n-1)\pi x}{a}\, dx$$

$$1 = \frac{B^2}{2}\left(x + \frac{a \sin \frac{2(2n-1)\pi x}{a}}{2(2n-1)\pi}\right)\Bigg|_{-a/2}^{a/2}.$$

$$1 = \frac{B^2}{2}\left(\frac{a}{2} + \frac{a}{2(2n-1)\pi} \sin \frac{\cancel{2}(2n-1)\pi \cancel{a}}{\cancel{2}\cancel{a}} + \frac{a}{2} - \frac{a}{2(2n-1)\pi} \underbrace{\sin \frac{-\cancel{2}(2n-1)\pi \cancel{a}}{\cancel{2}\cancel{a}}}_{0} \right)$$

$$1 = \frac{B^2}{2}\,\frac{\cancel{2}a}{\cancel{2}}$$

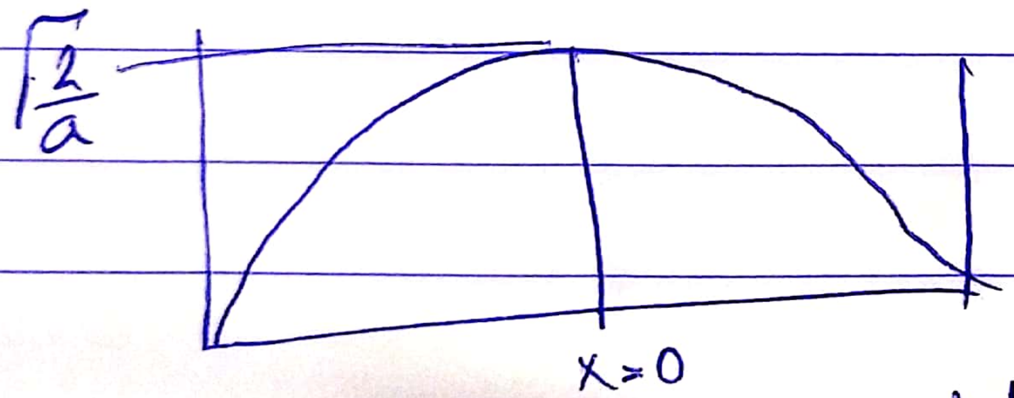
$$1 = \frac{B^2 a}{2}$$

$$\sqrt{\frac{2}{a}} = B$$

$$\Psi(x,t) = \left(\sqrt{\frac{2}{a}} \sin \frac{L\pi x}{a} \, e^{\frac{-iE_L t}{\hbar}} + \sqrt{\frac{2}{a}} \cos \frac{M\pi x}{a} \, e^{\frac{-iE_m t}{\hbar}} \right)$$

where $L \to$ even, $m \to$ odd.

The wave function of a particle in an infinite potential well is quantized, the function satisfy the boundary conditions imposed by the potential exist only for discrete values of the wave function leads to discreate energy.



at E = 1 $\quad E_L = \frac{\hbar^2 k_L^2}{2m}$

$E_L = \frac{L^2 \pi^2 \hbar^2}{2ma}$

← نعوض النظر L^2 او m^2
راح يطلع زوجي ،